Absis: Mathematics Education Journal
Vol. 7., No. 2, November 2025, pp. 67-77
ISSN 2686-0104 (print), 2686-0090 (online)
http://journal.univetbantara.ac.id/index.php/absis/index

Students' Errors in Exponential and Logarithmic Functions: An Analysis Using AVAEM Categories

Febi Tasya Ramadhanti [a,1*]

[a] Mathematics Education, Mulawarman University, Samarinda, Indonesia
[1] febitasya@fkip.unmul.ac.id*
* Corresponding Author

Received 13 October 2025; Accepted 02 December 2025; Published 31 December 2025

ABSTRACT

This study aims to analyze students' errors in exponential and logarithmic functions using the AVAEM error category framework: ARITH, VAR, AE, EQS, and MATH. A qualitative method with a case study approach was employed. Thirty-four tenth-grade students from a high school in Samarinda City were involved as research subjects. Data were collected through written tests and interviews with representatives from each error category. The identified AVAEM errors included: 1) ARITH—errors in understanding the rules of operations and properties of exponents and logarithms; 2) VAR—errors in understanding the meaning and role of variables; 3) AE—errors in seeing the structure of algebraic expressions; 4) EQS—misinterpreting the "=" sign as procedural instead of equality; and 5) MATH—errors in vertical and horizontal mathematization. These errors largely stem from difficulties in applying exponent and logarithm properties, memorizing formulas without conceptual understanding, and correctly modeling equations. Consequently, this study offers a valuable starting point for investigating students' learning obstacles, providing a basis for creating a didactical design to help teachers guide students in overcoming them.

KEYWORDS

Error Analysis
Exponential and Logarithmic Functions
AVAEM Categories

1. Introduction

Algebra is an essential branch of mathematics that plays a role in shaping students' mathematical thinking skills. The scope of algebra includes discussing concepts, simplifying expressions, and solving problems using symbols or variables. If students' understanding of these concepts is still low, they will tend to have difficulty solving algebra problems (Zulfa et al., 2020). In fact, algebra plays a vital role in generalizing and representing various problems through mathematical statements that describe the relationships between concepts. It serves as a universal language in multiple disciplines (Juliana et al., 2024). In addition, algebra serves as the foundation for many other branches of mathematics, such as calculus, geometry, trigonometry, matrices, vectors, and statistics (Jupri et al., 2014). Therefore, a good understanding of algebra is an important prerequisite for students to study advanced mathematics without encountering significant obstacles.

One of the important algebra topics to master in high school is the study of exponential and logarithmic functions. This topic requires students to master the properties of exponents and logarithms, operations with powers, and the concept of functions (Dwi et al., 2024). Although the concept is abstract, exponential and logarithmic functions have a wide range of applications, both in everyday life and in other fields of science (Saputro, 2019). A deep understanding of this topic is necessary because the concepts of exponential and logarithmic functions often reappear in advanced material, including inverse functions, growth and decay, and calculus.

Students' difficulties in understanding exponential and logarithmic functions largely stem from a weak grasp of basic material at the junior high school level, such as exponents and roots. Conceptual errors in this basic material, such as incorrectly applying the properties of exponents or adding numbers within roots, can persist into high school and directly impact errors in solving exponential and logarithmic problems (Putri et al., 2024). Based on an interview with the mathematics teacher who teaches the class involved in this study, many students still do not follow the rules of exponents because they do not understand the initial concepts, making it difficult for them to solve related problems. This condition emphasizes the importance of in-depth error analysis to reveal students' thinking patterns and the root causes of errors (Arigiyati et al., 2021; Mariani et al., 2023; Yodiatmana & Kartini, 2022).

The independent curriculum emphasizes the urgency of mathematics learning that is oriented towards understanding concepts and applying procedures in a meaningful way, not only in the context of real life, but also in the context of understanding the structure and mathematical nature of a concept (Hindri et al., 2023; Kemdikbudristek, 2022; Ndari et al., 2023). This approach requires students to understand the relationships between ideas in mathematics and be able to apply basic concepts appropriately to various forms of representation, including exponential and logarithmic functions. However, findings in the field indicate that this goal has not been optimally achieved, as many students still focus solely on the calculation process without understanding the conceptual meaning behind it.

Students' mistakes in solving problems are one indicator of low conceptual understanding. These mistakes generally stem from difficulties in understanding concepts, representing problems in mathematical models, and performing symbolic manipulations (Jupri et al., 2014). Previous studies have shown that students often have difficulty understanding algebraic expressions, operating with numbers in algebraic expressions, interpreting the meaning of the equal sign, and using variables correctly (Jupri et al., 2014; Siregar et al., 2023; Wardat et al., 2021). In line with this, many students still have difficulty understanding the concepts of exponents and the application of logarithms, resulting in conceptual and procedural errors when working on problems involving exponents and logarithms (Savitri Dharma Suarka & Sukjaya Kusumah, 2024; D. Ulfa & Kartini, 2021; Zaenuri & Astutiningtyas, 2023).

A more systematic analysis of these errors requires a clear analytical framework. Therefore, this study employs the AVAEM framework (ARITH, VAR, AE, EQS, MATH) to provide a structured and systematic basis for identifying the specific nature of students' algebraic errors. This framework, adapted from the error categories proposed by (Jupri et al., 2014) and extended to include the mathematization aspect as discussed by (Jupri & Drijvers, 2016). ARITH refers to errors arising from performing arithmetic operations within algebraic expressions, such as miscalculating basic operations or misapplying operational rules and properties. VAR involves misunderstandings of variables, including difficulties in interpreting them as unknowns, generalized numbers, placeholders, or changing quantities. AE captures obstacles related to interpreting or manipulating algebraic expressions, such as misreading their structure, expecting a numerical result when none is possible, or failing to view an expression as a coherent whole. EQS relates to misconceptions about the equal sign, particularly when students treat it merely as a signal to compute rather than as a symbol of equivalence between two expressions. Finally, MATH concerns difficulties in mathematization, including errors in translating situations into mathematical representations (horizontal mathematization) or reorganizing symbolic forms within the mathematical system (vertical mathematization). These categories provide a comprehensive framework for analyzing student thinking, allowing researchers to classify error patterns more precisely.

Several previous studies have examined student errors using the AVAE framework (ARITH, VAR, AE, EQS). Studies by (Jupri & Drijvers, 2016; Putri et al., 2024) show that students' greatest weakness lies in arithmetic errors (ARITH). Similar findings were reported by (Zulfa et al., 2020), who found that ARITH and AE errors dominated in algebraic fractions. However, most previous studies only used four categories of AVAE errors without considering the mathematization aspect (MATH). In fact, identifying and understanding students' difficulties from a mathematization perspective can provide better insight into students' algebra learning(Jupri & Drijvers, 2016), including in the context of exponential and logarithmic functions. Therefore, this study uses the AVAEM framework by adding the Mathematization (MATH) category to analyze students' errors more comprehensively in the material on exponential and logarithmic functions.

Based on the above description, this study aims to analyze in depth the errors students encounter in exponent and logarithm material using the AVAEM error category framework, which includes five categories: ARITH (Applying Arithmetic Operations), VAR (Understanding the Notion of Variable), AE (Understanding Algebraic Expressions), EQS (Understanding the Meaning of the Equal Sign), and MATH (Mathematization) (Jupri et al., 2014). The results of this study are expected to provide a comprehensive overview of students' error patterns and serve as a reference for teachers in designing more effective learning strategies that are adaptive to students' needs.

2. Method

This study employed a qualitative method with a case study approach, as this approach provides a deep understanding of the phenomenon being studied and is well-suited for exploring students' thinking processes (Creswell & Creswell, 2018). Case studies were chosen to focus the analysis on students' errors in solving exponential and logarithmic functions in context. The research subjects were 34 tenth-grade students from a high school in Samarinda who had learned about exponential and logarithmic functions. Participants were selected based on their academic heterogeneity so that a wider variety of errors could be identified. The codes S1–S5 are pseudonyms used to maintain participants' confidentiality.

The research instrument consisted of a four-item essay test on exponential and logarithmic functions. The items were reviewed by two experts to ensure content validity and clarity. Before being used in the main study, the test was piloted with a group of students who possessed characteristics comparable to the research participants. The pilot analysis assessed item difficulty, discrimination index, and reliability, and the results indicated that all four items had moderate difficulty, adequate discriminatory power, and high reliability. Therefore, all items were considered valid and suitable for use in this study. In completing the test, students were required to write down the entire solution process without the aid of a calculator so that researchers could trace the thought process underlying their answers. In addition, unstructured interviews were conducted with several students who showed representative error patterns, aiming to strengthen and validate the results of the written test data analysis. The data analysis process referred to the steps of qualitative data analysis, which included data reduction, data presentation, and conclusion drawing (Miles et al., 2019). In the data reduction stage, all student answers were corrected to identify various types of errors in solving exponential and logarithmic functions. Each student's answer was grouped based on similar error characteristics to obtain data relevant to the research focus. Next, the data presentation stage was carried out by describing the results of the error classification using the AVAEM (Arithmetic, Variable, Algebraic Expression, Equation Sign, Mathematization) framework as developed by (Jupri et al., 2014), descriptively in the form of a narrative explaining the patterns of student errors. The conclusion-drawing stage was carried out by interpreting error patterns and their causes based on the results of test analysis and student interviews. Students were interviewed to explore how they solved problems and to confirm the suspected error patterns identified from the test results.

3. Result and Discussion

This study reveals students' errors in solving exponential and logarithmic function problems based on test results and interviews. The results of the study reveal five categories of errors identified in students' work on exponent and logarithm problems, namely ARITH, VAR, AE, EQS, and MATH (AVAEM). The researchers also found that some students made more than one category of error in their answers.

3.1. Errors in Arithmetic Operations (ARITH)

ARITH category errors are errors that indicate students' limitations in three aspects, namely 1) ability in using symbolic expressions such as addition, subtraction, multiplication, and division; 2) application of priority rules in arithmetic calculations, both in numerical and algebraic forms; 3) the application of numerical operation properties (commutative, associative, and distributive) (Jupri et al., 2014). The author found several student answers that contained errors in the ARITH category. Figure 1 shows the answer to question number 1 by student S1.

Jika x_1 dan x_2 akar-akar dari persamaan $3^{x+1} + \frac{1}{3^{x-2}} = 28$, maka tentukan nilai dari ${x_1}^2 + {x_2}^2$.

Translation:

If x_1 and x_2 are the roots of the equation $3^{x+1} + \frac{1}{3^{x-2}} = 28$, then determine the value of ${x_1}^2 + {x_2}^2$.

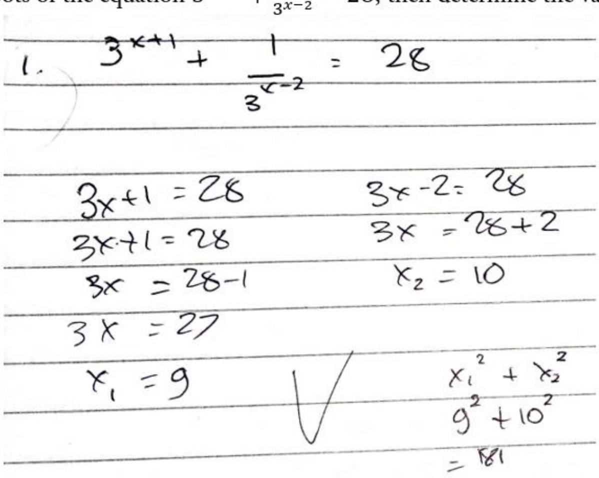

Fig. 1. S1's answer to Question 1

Jika x_1 dan x_2 akar-akar dari persamaan $3^{x+1} + \frac{1}{3^{x-2}} = 28$, maka tentukan nilai dari ${x_1}^2 + {x_2}^2$.

Translation:

If x_1 and x_2 are the roots of the equation $3^{x+1} + \frac{1}{3^{x-2}} = 28$, then determine the value of ${x_1}^2 + {x_2}^2$.

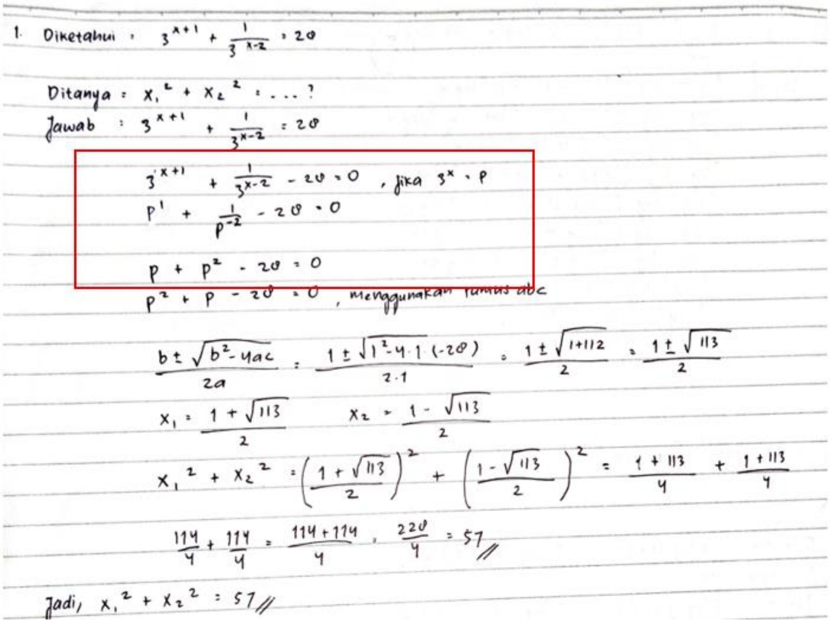

Fig. 2.S2's answer to Question 1

In Figure 1, students interpret exponential expressions as linear operations. The students' answer shows that they have misapplied the rules of exponential operations. The form 3^{x+1} is interpreted as $3x + 1$ and $\frac{1}{3^{x-2}}$ is interpreted as $3x - 2$. This is a fundamental error in treating exponential notation, where the exponent does not match the multiplication between the base number and the variable. This error falls under the ARITH category, which relates to the rules of exponent operations, because the student treats the exponent as a simple multiplication between a number and a variable. The following excerpt from an interview with the student shows that they genuinely do not understand the rules of exponent properties.

R = Researcher

R : "Please explain to me how you solved this problem."

S1 : "I don't understand, Miss. I didn't know the answer, so I just filled in the blanks. I tried to solve it like this. I wrote $3x + 1 = 28$, then moved the terms to get $x_1 = 9$. Similarly, for $3x - 2 = 28$, after moving the terms, I got $x_2 = 10$. Next, I calculated the value of $9^2 + 10^2$, Miss."

R : "Are you sure about your answer?"

S1 : "No, Miss, I don't really understand."

R : "Do you understand the properties of exponents? (while writing) If $a^x \times a^y = \cdots$?"

S1 : "(while writing) So it's a^{xy}, Miss? I don't know if it is correct or not."

R : "Then (while writing) if $a^x : a^y = \cdots$?"

S1 : "Hmm, I don't know, Miss. I forgot."

The student did not answer using the correct exponent rules. From the very beginning, the student made a mistake, which led to incorrect subsequent answers. This was because the student did not understand the concept of exponents. The student's answers also contained other types of errors, which will be discussed in the next section. The error in the ARITH category for question number 1 also occurred in the work of student S2 below.

In the student's answer marked in the red box, the student made a mistake in applying the exponent rule. The student incorrectly interpreted 3^{x+1} as p^1, when in fact, with the substitution $3^x = p$ the correct result is 3p. Similarly, with $\frac{1}{3^{x-2}}$, the student also incorrectly interpreted it as p^2. Based on the interview results, the student was able to state the properties of exponents in the form correctly

$a^x \times a^y = a^{x+y}$ and $a^x : a^y = a^{x-y}$. They were still confused when applying these properties to answer questions. The cause of this error was that the student only memorized the formula without understanding the concept of the exponent rule itself.

R : "Please explain to me how you solved it."

S2 : "First, I assumed that $3^x = p$. So I changed 3^{x+1} to p^1 and $\frac{1}{3^{x-2}}$ to $\frac{1}{p^{-2}}$. Then I found the roots using the abc formula."

R : "Are you sure about your answer?"

S2 : "I am not sure, Miss."

R : "Do you understand the properties of exponents? If $a^x \times a^y = \cdots$?"

S2 : "a^{x+y}, Miss."

R : "Okay, then if $a^x : a^y = \cdots$?"

S2 : "a^{x-y}, Miss."

R : "Take another look at the exponential equation in question number 1. After assuming $3^x = p$, is $3^{x+1} = p^1$?"

S2 : "Ohhh yes, Miss, my previous answer was wrong. It should be $3^x \times 3^1$."

Furthermore, an ARITH error was also found in the S3 student's answer to question number 3, as shown in Figure 3. An interview was also conducted to clarify the results of the student's work.

R : "Please explain to me how you solved this?"

S3 : "I used the logarithm property, Miss. I simplified the right side $^2 \log 2 + ^2 \log x = ^2 \log 2x$. Then I got confused, Miss."

R : "Okay, then, why did $2^{x+1} + 15$ disappear on the left side and change to $^2 \log 2^2 + ^2 \log 2^{15}$?"

S3 : "I forgot, Miss. I don't remember how I got it now. When I was working on it, I scribbled on another piece of paper and got that result."

R : "Are you sure about the answer?"

S3 : "I'm not sure, Miss. I'm not sure if my first step is correct. I think it's wrong. If the first step is wrong, then the next steps must be wrong too, Miss."

Tentukan himpunan penyelesaian dari $^2\log {}^2\log(2^{x+1} + 15) = 1 + ^2\log x$.
Translation:
Determine the set of solutions of $^2\log {}^2\log(2^{x+1} + 15) = 1 + ^2\log x$.

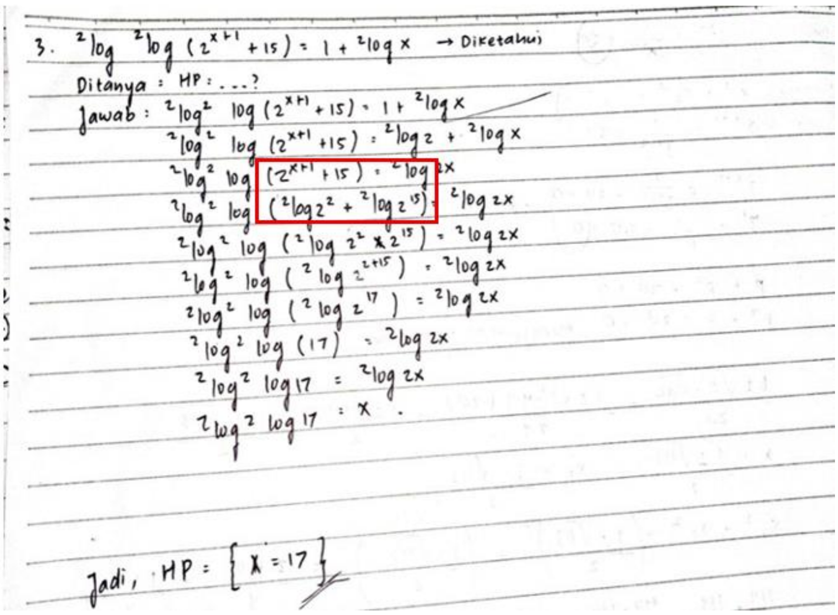

Fig. 3. S3's answer to Question 3

Based on the interview results, the student made a mistake from the start and already felt that the answer was wrong. This error also falls under the ARITH category because the student misapplied the properties of logarithms. This is evident in the student's answer marked in the red box, where the student

Absis: Mathematics Education Journal
Vol. 7., No. 2, November 2025, pp. 67-77

thought that the logarithm of a sum could be broken down into the sum of logarithms. This error is not merely a calculation error, but a misunderstanding of the fundamental properties of logarithms.

These findings are consistent with previous research showing that arithmetic errors in algebraic expressions mostly stem from students' tendency to use the strategy of "memorizing procedures" without understanding the underlying mathematical concepts. These include difficulties in recognizing exponential patterns and errors in operations related to exponents, such as multiplication and division of exponential numbers (Putri et al., 2024; Suarka & Kusumah, 2024). Therefore, learning interventions that emphasize understanding the meaning of exponents and logarithms (rather than just the rules of calculation) are needed to minimize ARITH-type errors. By linking specific examples of errors in exponent-logarithm problems to the ARITH category, this study demonstrates how arithmetic errors in exponents and logarithms impact the overall problem-solving process.

3.2. Errors in Understanding Variables (VAR)

The next mistake made by students is a VAR category error. Students who have difficulty in the VAR category can be seen from their inability to understand variable notation, in the following conditions: 1) interpreting symbols only as single values rather than a set of values; 2) replacing a literal symbol in an equation with a specific value, and getting the wrong result. (Jupri et al., 2014). The students' answers in Figure 2 are also errors classified as VAR errors, namely, errors in understanding the meaning of variables or symbol substitution. Students treat the solution of the quadratic equation (value p) as if it were the value x without returning the substitution ($x = {}^{3}\log p$). Students are confused between the substitute variable p and the original variable x.

R : "After obtaining the quadratic equation in the form of p, then you find the roots of the quadratic equation using the abc formula, then what do you do?"

S2 : "I continue to find the value of x, then write the conclusion first, Miss. After that, I work on question number 2."

R : "Did you double-check your calculations? Why did the roots of the quadratic equation suddenly appear in the variable x, even though the quadratic equation had a variable p?"

S2 : "No, Miss, because I wanted to go home early, and I did not really know how to do it. I was confused."

The classification of this student's error into the VAR category is reinforced by the interview results, which indicate that the student still has difficulty understanding the role of variables. Although the student was mistaken in creating a quadratic equation in a variable p (ARITH category error), from the student's thought process, as stated in the answer and interview results, it is clear that the student did not understand that after finding the solution for the substitute variable p, it must be returned to the variable x through $p = 3^x \Rightarrow x = {}^{3}\log p$, or check whether the solution $p > 0$, and then take the log.

The findings of this study corroborate previous findings showing that differences in understanding between concrete numbers and symbols (variables) are often a source of algebraic errors, particularly when students switch from numerical to symbolic representations and vice versa (Chan et al., 2022; Putri et al., 2024; N. Ulfa et al., 2024). Therefore, it is important to include exercises to familiarize students with substituting values for variables and to double-check at the end of the problem. This study demonstrates that errors in the VAR category can hinder the process of solving exponential and logarithmic function problems, and confirms that understanding variables in algebra remains a significant challenge at the high school level that requires more attention from teachers.

3.3. Errors in Algebraic Expressions (AE)

The following error is the AE category error, which includes various obstacles students encounter when dealing with algebraic expressions. These obstacles involve understanding the structure, meaning, and formal rules of writing, as well as simplifying algebraic expressions. There are four main types of difficulties. First, the parsing obstacle, which is students' confusion due to the difference between the order of natural language and algebraic language. Second, the expected answer obstacle occurs when students always expect to get an answer in the form of a number, even though the solution is an algebraic expression. Third, the obstacle of a lack of closure, which is the tendency of students to add or subtract numbers with algebraic terms, resulting in an incorrect form. Fourth, lack of gestalt view, which is when students ignore the integrity of an expression's structure, such as omitting inequality signs, positive/negative signs, or variables when performing algebraic manipulations (Jupri et al., 2014). Figure 4 presents examples of students' answers that fall into the AE error category.

Tentukan himpunan penyelesaian dari ${}^{\frac{1}{5}}\log(x + \sqrt{3}) + {}^{\frac{1}{5}}\log(x - \sqrt{3}) < 0$

Translation:

Find the solution set of ${}^{\frac{1}{5}}\log(x + \sqrt{3}) + {}^{\frac{1}{5}}\log(x - \sqrt{3}) < 0$

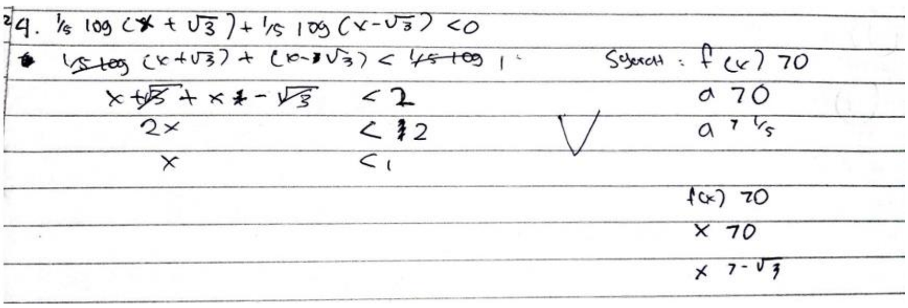

Fig. 4. S4's answer to Question 4

The category of student errors in Figure 4 falls under AE-Lack of gestalt, which is the failure to see the whole structure of logarithmic expressions. Students ignore the fact that both logarithmic terms have the same base, allowing them to be combined into a single logarithm with the product of the arguments. They see the logarithmic expression only as a "number" that can be added directly to the argument, thus losing the overall meaning of the expression.

Students view the two terms ${}^{\frac{1}{5}}lo\,g(x + \sqrt{3}) and\ {}^{\frac{1}{5}}lo\,g(x - \sqrt{3})$ as separate "numbers," not as part of a logarithmic structure that has specific rules for combining: ${}^{a}lo\,g\,M + {}^{a}lo\,g\,N = {}^{a}lo\,g(M.N)$. Ignoring this, students immediately add the logarithm arguments in an invalid way — even replacing the second logarithm with its argument alone. In addition, students also ignore the special properties of logarithms with base $a = \frac{1}{5} < 1$, where the direction of the inequality should be reversed when the logarithm is removed. This shows that students do not see the expression as a logarithm with specific rules and behavior, but rather process it mechanically. The following are the results of interviews with students:

R : "After understanding the question, what did you do next?"

S4 : "I tried to solve it, so that the base on the left side would be the same as the base on the right side. I changed the 0 on the right side to ${}^{\frac{1}{5}}\log 1$. Then, I just crossed it out. I thought it could be crossed out. I was confused about how to continue, so I just tried adding $(x + \sqrt{3})$ and $(x - \sqrt{3})$."

R : "Then what does $a > 0$ mean?"

S4 : "A means the result, I think it's wrong, Miss, I'm confused."

This finding is in line with the interview, in which the student explained that he "crossed out" ${}^{\frac{1}{5}}\log 1$, because he thought it was equal to zero, then tried to add $(x + \sqrt{3})$ to $(x - \sqrt{3})$ because he was confused about how to continue the solution. Even when asked about the condition $a > 0$, the student interpreted "a" as the final result, not as the logarithm base. Thus, both from the written work and the interview, it is clear that the student had difficulty understanding the logarithm structure as a whole and failed to apply the applicable formal rules.

The results of the study indicate that AE errors are often structural in nature, meaning that students do not view algebraic expressions, particularly logarithms, as a unified whole, but instead process parts mechanically. This is in line with the results of a study of misconceptions in logarithms by (Díaz-Berrios & Martínez-Planell, 2022), which stated that logarithm misconceptions occur because students fail to see exponents and logarithms as an interconnected system. This finding confirms that AE errors are not simply errors in calculation steps, but arise from students not understanding the meaning and structure of algebra in depth. Furthermore, this study shows that the AE error category does not only appears in general algebraic manipulations but is also very prominent when students are faced with exponential-logarithmic structures.

ISSN 2686-0104 (print), 2686-0090 (online)

Vol. 7., No. 2, November 2025, pp. 67-77

3.4. Errors in Interpreting the Equal Sign (EQS)

The EQS category refers to students' errors in understanding the meaning of the "=" sign in algebra or arithmetic. This error arises because students do not perceive the equal sign as an equivalence relation indicating that two mathematical forms have the same value; instead, they treat it merely as a procedural sign to continue the operation (Jupri et al., 2014).

The students' answers in Figure 1 also contain the EQS error category. This can be seen where students write a series of equations that appear to be equivalent: $3x + 1 = 28$ and $3x - 2 = 28$, even though these expressions are not equivalent to the original question $3^{x+1} + \frac{1}{3^{x-2}} = 28.$ This means that students do not actually use the "=" sign to express algebraic equality, but rather as a "running statement." Equality is not maintained; instead, it is written in a chain as if all lines are correct. In algebra, the "=" sign means that two expressions have the same value. In this case, students use it inconsistently: they replace the exponential form 3^{x+1} with the linear form $3x + 1$, then continue to write the "=" sign. Students do not understand the meaning of the equal sign as algebraic equivalence, but merely as a sign to "continue the calculation."

In this category of EQS errors, the author selected two student answers to different questions. The student's answer to question number 3, as shown in Figure 3, also contains an EQS error, because the equal sign does not maintain equivalence. After incorrectly breaking down the logarithmic form, the student continues to write the "=" sign as if the steps were equivalent. In fact, the resulting equation has a different meaning from the original question.

This finding is reinforced by research by (Wardat et al., 2021), which revealed that students who do not yet understand the meaning of equivalence often perform incorrect algebraic manipulations, even though the procedure appears correct. These EQS errors indicate that students do not yet realize that each solution step must be algebraically equivalent to the original form. As a result, their subsequent steps are invalid, disrupting the entire solution process and can even lead to more complex errors in advanced materials. Therefore, it is important for teachers to re-emphasize the meaning of the equals sign as an equivalence relation and to familiarize students with checking whether each manipulation maintains this equivalence.

3.5. Errors in Mathematization (MATH)

The last student error that was successfully identified was in the MATH subject. The MATH category refers to student errors in performing mathematization, both horizontally and vertically (Jupri et al., 2014).

Tentukan himpunan penyelesaian pertidaksamaan eksponen $2\sqrt{4^{x^2-3x+2}} < \sqrt[3]{\frac{1}{2}^{3-6x}}$

Translation:

Determine the solution set of $2\sqrt{4^{x^2-3x+2}} < \sqrt[3]{\frac{1}{2}^{3-6x}}$

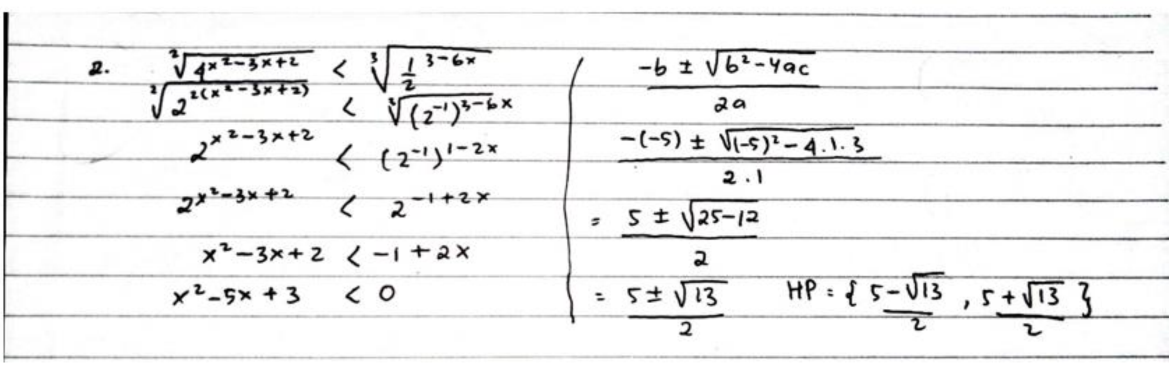

Fig. 5. S5's answer to Question 2

The students' errors in Figure 5 fall into the category of MATH errors, both horizontal and vertical mathematization. In horizontal mathematization errors, the model created by the students is incorrect in simplifying the initial form. When changing the left side to a simpler form, the students changed important information (the multiplier 2). This makes the mathematical model that is solved no longer equivalent to the original problem.

Then, in vertical mathematization, it appears that the symbols are manipulated correctly, but the model is wrong. After incorrectly modeling, specifically by forming an incorrect power-based inequality, the student solves the quadratic inequality using the correct procedure. The student correctly performs the following steps: equating the bases 2, comparing the exponents, solving the quadratic $x^2 - 5x + 3 < 0$ using the abc formula, and finding the solution set as the intersection of two intervals. However, all of this was done on an incorrect model from the start, so the solution set obtained was not the correct solution set for the problem. This means that the algebraic process was technically correct, but irrelevant because the initial equation was incorrect.

R : "Do you think the number 2 on the left side indicates a square root?"
S5 : "Oh, yes, is that 2 times the root or a square root? What is it? Well, I think it is a square root, Miss."
R : "What did you do next?"
S5 : "I did not really understand how to do it, Miss, so I tried it myself. I tried to remove the root form so that both sides would be in exponential form."
R : "Okay, please explain to me how you solved it."
S5 : "So on the left side, it is 4, Miss. I changed it to 2^2. Similarly, on the right side, I changed $\frac{1}{2}$ to 2^{-1}. Then I tried to remove the roots. On the left side, it is a square root, and on the right side, it is a cube root, Miss."
R : "Then what is the next step?"
S5 : "After simplifying, the right and left sides are both 2 to some power, so I cross out the number 2, leaving the power. Then I find x_1 and x_2 using the abc formula."
R : "What happens next?"
S5 : "In my opinion, the solution set becomes $\frac{5-\sqrt{13}}{2}$ and $\frac{5+\sqrt{13}}{2}$, Miss."
R : "Are there only two values of x that satisfy this inequality?"
S5 : "I do not know, Miss."

These findings suggest that the ability to execute algebraic problem-solving procedures alone is insufficient. If the initial model a student constructs is inaccurate, the entire subsequent process becomes irrelevant to the given problem. Studies (Aguirre et al., 2024; Maass et al., 2023) show that successful problem-solving depends on the ability to translate context into a mathematical model correctly and to maintain symbol manipulation consistent with the model. Therefore, preventing MATH errors requires not only reinforcing algebraic procedures but also guiding students in constructing and verifying their mathematical models. This research demonstrates that difficulties in mathematical modeling are also a significant source of error and should be analyzed to provide a more comprehensive picture of student difficulties. To date, the AVAE approach has emphasized procedural errors and symbol manipulation. By adding the MATH category to the analysis of exponent-logarithm material, this study broadens the scope of error identification to include modeling aspects, thereby providing a more comprehensive understanding of student error in solving.

4. Conclusion

This study aimed to analyze students' errors in solving exponential and logarithmic function problems using the AVAEM framework. The findings show that the five AVAEM categories—ARITH, VAR, AE, EQS, and MATH—offer a clear structure for identifying the fundamental sources of students' errors. The analysis indicates that these errors arise not only from procedural lapses but also from deeper conceptual weaknesses, particularly students' limited understanding of exponent and logarithm properties. Many students rely on memorized rules without grasping the underlying concepts, leading them to construct incorrect initial models; as a result, even technically correct algebraic steps can produce incorrect final answers because the starting point is flawed. By mapping these difficulties systematically, the study provides insights that teachers can use to design instructional strategies aimed at strengthening students' conceptual understanding and reducing recurring errors. This study can serve as a starting point for further exploration of student learning obstacles within the AVAEM framework and may inform the development of didactic designs that help students overcome these obstacles, offering meaningful prospects for improving mathematics learning.

References

Aguirre, J. M., Turner, E. E., Mcvicar, E., Mcduffie, A. R., Foote, M. Q., & Carll, E. (2024). *Mathematizing the World: A Routine to Advance Mathematizing in the Elementary Classroom*. https://www.sciencedirect.com/science/article/pii/S0732312324000737

Arigiyati, T. A., Yumarsa, W., Novianti, E. W., & Kusumaningrum, B. (2021). Students Error Analysis In Exponential And Logarithmic Equations. *Hipotenusa : Journal of Mathematical Society*, *3*(2), 158–170. https://doi.org/10.18326/hipotenusa.v3i2.6312

Chan, J. Y. C., Ottmar, E. R., Smith, H., & Closser, A. H. (2022). Variables Versus Numbers: Effects of Symbols and Algebraic Knowledge on Students' Problem-Solving Strategies. *Contemporary Educational Psychology*, *71*. https://doi.org/10.1016/j.cedpsych.2022.102114

Creswell, J. W., & Creswell, J. D. (2018). *Research Design: Qualitative, Quantitative, and Mixed Methods Approaches* (5th ed.). SAGE Publications.

Díaz-Berrios, T., & Martínez-Planell, R. (2022). High School Student Understanding of Exponential and Logarithmic Functions. *The Journal of Mathematical Behavior*, *66*, 100953. https://doi.org/10.1016/j.jmathb.2022.100953

Dwi, Y., Muhammad, K., Al, T., Masta, A., Yulfiana, E., & Revisi, E. (2024). *Matematika Tingkat Lanjut untuk SMA/MA Kelas XI (Edisi Revisi)*. Pusat Perbukuan Kompleks Kemdikbudristek. https://buku.kemdikbud.go.id

Hindri, S. H., Sutisnawati, A., & Lyesmaya, D. (2023). Learning Analysis: Implementation of the Independent Curriculum in Improving Numeracy Literacy in Elementary School. *EDUHUMANIORA: Jurnal Pendidikan Dasar Kampus Cibiru*, *15*(2), 173–182. https://doi.org/10.17509/eh.v15i2.57200

Jupri, A., & Drijvers, P. (2016). Student Difficulties in Mathematizing Word Problems in Algebra. *Eurasia Journal of Mathematics, Science and Technology Education*, *12*(9), 2481–2502. https://doi.org/10.12973/eurasia.2016.1299a

Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in Initial Algebra Learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683–710. https://doi.org/10.1007/s13394-013-0097-0

Kemdikbudristek. (2022). *Panduan Pembelajaran dan Asesmen: Kurikulum Merdeka*. https://kurikulum.kemdikbud.go.id/wp-content/uploads/2022/06/Panduan-Pembelajarn-dan-Asesmen.pdf

Maass, K., Zehetmeier, S., Weihberger, A., & Flößer, K. (2023). Analysing Mathematical Modelling Tasks in Light of Citizenship Education Using the COVID-19 Pandemic as a Case Study. *ZDM - Mathematics Education*, *55*(1), 133–145. https://doi.org/10.1007/s11858-022-01440-9

Mariani, M., Hutapea, N. M., & Maimunah, M. (2023). Analysis Of Students' Mistakes In Completing Exponent Materials Based On Castolan Theory. *Journal of Medives : Journal of Mathematics Education IKIP Veteran Semarang*, *7*(1), 135. https://doi.org/10.31331/medivesveteran.v7i1.2333

Miles, M. B., Huberman, A. M., & Saldaña, J. (2019). *Qualitative Data Analysis: A Methods Sourcebook* (4th ed.). SAGE Publications.

Ndari, W., Suyatno, Sukirman, & Mahmudah, F. N. (2023). Implementation of the Merdeka Curriculum and Its Challenges. *European Journal of Education and Pedagogy*, *4*(3), 111–116. https://doi.org/10.24018/ejedu.2023.4.3.648

Putri, C. K., Prabawanto, S., & Juandi, D. (2024). Students' Errors in Solving Powers and Roots Based on AVAEM (ARITH, VAR, AE, EQS, and MATH) Categories. *KnE Social Sciences*. https://doi.org/10.18502/kss.v9i13.16075

Saputro, A. P. (2019). *Matematika untuk Kehidupan: Fungsi Eksponensial*. Deepublish.

Savitri Dharma Suarka, N. K., & Sukjaya Kusumah, Y. (2024). Students' Learning Obstacles in Exponential: A Case Study in Indonesian Higher Education Students. *Jurnal Pendidikan MIPA*, *25*(2), 530–541. https://doi.org/10.23960/jpmipa/v25i2.pp530-541

Siregar, R., Siagian, M. D., & Wijaya, T. T. (2023). Exploration of Students' Epistemological Obstacles in Understanding the Concept of Variables and Expressions. *Jurnal Didaktik Matematika*, *10*(1), 63–76. https://doi.org/10.24815/jdm.v10i1.30694

Suarka, N. K. S. D., & Kusumah, Y. S. (2024). Students' Learning Obstacles in Exponential: A Case Study in Indonesian Higher Education Students. *Jurnal Pendidikan MIPA*, *25*(2), 530–541. https://doi.org/10.23960/jpmipa/v25i2.pp530-541

Ulfa, D., & Kartini, K. (2021). Analisis Kesalahan Siswa dalam Menyelesaikan Soal Logaritma Menggunakan Tahapan Kesalahan Kastolan. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, *05*(01), 542–550. https://doi.org/https://doi.org/10.31004/cendekia.v5i1.507

Ulfa, N., Jupri, A., & Juandi, D. (2024). Error Analysis on the Concept of Composition of Function in Online Learning from the Perspective of AVAE Categories (ARITH, VAR, AE, and EQS). *KnE Social Sciences*. https://doi.org/10.18502/kss.v9i8.15620

Wardat, Y., Jarrah, A. M., & Stoica, G. (2021). Understanding the Meaning of the Equal Sign: A case Study of Middle School Students in the United Arab Emirates. *European Journal of Educational Research*, *10*(3), 1505–1514. https://doi.org/10.12973/EU-JER.10.3.1505

Yodiatmana, Y., & Kartini, K. (2022). Analysis of Student Errors in Solving Basic Logarithmic Problems Using Kastolan Error Analysis. *Jurnal Gantang*, *7*(2), 129–136. https://doi.org/10.31629/jg.v7i2.4689

Zaenuri, M. D., & Astutiningtyas, E. L. (2023). Analisis Kesalahan Siswa Dalam Menyelesaikan Soal Logaritma Pada Siswa Kelas X PPG SMK Muhammadiyah 1 Sukoharjo. *JP2M (Jurnal Pendidikan Dan Pembelajaran Matematika)*, *9*(1), 20–34. https://doi.org/10.29100/jp2m.v9i1.3605

Zulfa, B. I., Suryadi, D., Fatimah, S., & Jupri, A. (2020). Student's Mistake in Algebraic Fraction: An Analysis Using AVAE Categories. *Journal of Physics: Conference Series*, *1521*(3). https://doi.org/10.1088/1742-6596/1521/3/032029